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Social Norms and Conventions as Coordination Devices of Behavior Choices among Agents ; Overview

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1. Introduction

As the world economy is integrated and as economies with different styles from Western countries, such as post-war Japan and Korea, attain high economic performance, the importance of the study on diversity of economies is recognized. In the past decade, the diversity of economic systems observed among countries or between different points of time is studied theoretically. This line of study has been developed in a systematic way and called Comparative Institutions Analysis (CIA). A lot of literature in CIA explains the rationale hidden behind the Japanese management system which has not been satisfactorily understood, e.g., "lifetime employment" and "enterprise unionism" in the employer-employee relationship, "seniority pay" in wage structure and promotion system and "mutual stockholding" in corporate governance (as an extensive survey, see Aoki (1995)).

Aoki (1996) claims the importance of analyzing the economy as a system consisting of mutually complementary factors. By doing so, we can think that these arrangements have the rationality in that they are equilibrium strategies supporting each other, then there is no incentive for agents to change only one of them. And he regards not only the arrangements listed above but also market, money, legal and political ruling by state, contracts, social conventions and norms as the institutional arrangements supported as mutually complementary equilibrium strategies. But we distinguish the institutional arrangements which are the obligatory constraints to the behavior of agents which cannot change without formal legal procedure from those which can be chosen by any single agent under these constraints. We call the latter informal constraints for agents social norms and conventions.

The main purpose of this paper is to study why and how such diversity emerges and such variety kinds of social conventions and norms are maintained over time. In the conventional studies in CIA, such diversity is regarded as a consequence of a variety of historically given physical conditions like endowments, production technology and preference. On the other hand, in our analysis, we regard it as a consequence of coordination of choices among agents and as being determined by economic performance, expectation and stock of information given physical conditions.[1] That is, we regard the relationship among agents and that among organizations or societies as being constraints on decision-making of agents. To this end, we take a game theoretical approach. According to Schelling (1960), game theory is the theory of the inter-dependence decision which ranges over a spectrum with games of pure conflict and games of pure coordination as opposite limits.[2] A typical example of the former is well known as "prisoner's dilemma" game in which defection strictly dominates cooperation for both players. We shall be concern with exactly or nearly pure coordination problems.

This paper consists of four sections. And the outline of each section is as follows. Section 2 and 3 are devoted to comparison of interpretations of social convention and norm by economists and game theorists with that by other social scientists. A social convention and norm have been studied not only by economists but also other social scientists and there has been much difference between their definitions. Specifically, economists regards economic phenomena observed in the real economy as consequences of agents' choice subject to physical and institutional constraints, e.g., endowment, technology, market power and so on. And they interpret social conventions and norms in the same way. On the other hand, other social scientists define it as orientations by which individuals are guided. That is, they interpret them to be constraints on agents' decision-making. We call the former "equilibrium analysis" and the latter "philosophycal study".

Section 4 and 5 are devoted to presentation of two analytical tools which play the key role in our analysis. First one is the Folk Theorem in the theory of repeated games. This states that efficient cooperation can be supported as an equilibrium outcome in non-cooperative games.

Second one is the concept of evolutionarily stability in the theory of evolutionary game. The fact that evolutionarily stable equilibrium is Nash equilibrium is called by Samuelson and Zhang (1992) and Samuelson (1994)

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1 Throughout this paper, we assume that all agents are boundedly rational in the sense defined. Then the entire history and the whole structure of economy are not supposed to be common knowledge and in decision-making they exchange necessary information and form expectation about economic performance of their choice.

2 See Schelling pp.83-118, 291-303.

"stability or convergence implies Nash theorem" and a version of the Folk Theorem of evolutionary games.

Section 6 is devoted to presentation of the key concept in our analysis which is "complementarity" . Lastly, section 7 is devoted to pointing out some problems in the conventional analysis and presentation of the outline of our analysis in the following chapters.

2 A Social Norm ; Philosophycal Study and Equilibrium Analysis

2.1 A Social Norm

In this subsection, we discuss how a social norm should be interpreted in our analysis and how it has been sustained by the member of the society. We begin with discussing the first question. Elster(1989) and Matui (1996) argue that in the social sciences, different disciplines have different definitions of social norm and there is discrepancy between the interpretation of social norm by economists and that by sociologists. On the one hand, sociologists define it as an orientation by which individuals are guided. And this orientation becomes a social norm when it is shared by most members in a society. It is interpreted as a constraint on agents' decision-making and usually induces an inefficient outcome.

Then, why do agents obey the norms in spite of their inefficiency. Elster(1989) gives a reason for it with the following two concepts, external and internal sanctions.[3] He argues that norm-guided behavior is supported by the threat of social sanctions that make it rational to obey the norms. And the sanctions are performed because of the fear of being sanctioned. However, in many cases, agents conform to norms even if their violation of norms is unobserved and not exposed to sanctions because they have feelings of guilt, anxiety and shame. For the reason, they would internalize the norms and obey them. If the punishment is only a label attached to the deviant, nobody will feel shame when one is caught. This observation implies that agents are

[3] See pp.104-105.

not engaged in so called cost-benefit analysis, nor do they pay too much attention to the action's consequence. These feelings can be sufficient internal sanction to make them follow a social standard independently of the current reaction of others.

On the other hand, economists sometimes define it as an equilibrium of games describing the economic activities in the society. Economists regards economic phenomena observed in the real economy as consequences of agents' choice subject to physical and institutional constraints. And they usually assume rational (or at least boundedly rational) agent whose behavior choice is made in a goal-oriented way and always in his self-interest. As a consequence, they examine the rationale hidden behind the social norm to which all agents conform. Economists pointed out that norms have the following functions in the society.

(1) It helps agents to achieve individual or social welfare.
(2) It helps agents to prevent market failure and coordination failure.
(3) It helps agents to economize on decision cost.

The difference between two interpretations discussed here can be simply summarized in ***Figure 1.***

	Philosophycal Study	**Equilibrium Analysis**
Agent	**Not Rational**	**Rational (or at least Boundedly Rational)**
Behavior	**Norm-Guided**	**Goal-Oriented**
Social Norm	**Constraint**	**Outcome (Equilibrium)**
Efficiency	**Usually Inefficient**	**Usually Efficient**

Figure 1.

Many economists have made an attempt to bridge the gap by establishing either that social norms are rational as efficient means to achieve individual and social welfare, or that social norms make it rational to confirm to them by giving agents incentive to do so. The latter attempt made by Kandori (1992) and Okuno-Fujiwara and Postalewaite(1995) is presented in the following subsection.

2.2 A Social Norm as an Coordination Mechanism among Agents in Games

In this subsection, we present a model where following the prevailing standard behavior can be equilibrium attained by agents who pursue their own self-interest and whose behavior are goal-oriented. That is Kandori (1992) and Okuno-Fujiwara and Postalewaite(1995)'s social norm model which defines a social norm as a combination of a transition mapping of agent's status level and a social standard of behavior corresponding to the status level. If the status level is changed based on his behavior choice in the previous period in the way that the low status level is attached to those who deviate from the prevailing standard behavior and the high level to those who follow it, a transition mapping works as an information processing mechanism for transmitting information about agents' past deviations. And if the social standard behavior prescribes defect for those who meet those who deviate from the social standard and cooperation for those who meet those who follow it, the standard behavior works as an enforcing mechanism of standard behavior and facilitates coordination among them through community enforcement. Community enforcement is a way of enforcing a certain social agreement by using punishment by all members of society who are not necessarily the victims of an agent's deviation. It does not require the whole structure of game and the entire history of play to be common knowledge because a label attached to each agent represents his behavior choice in the previous period.

Under these mechanisms, it is shown that the Folk Theorem holds in random matching game (for the detail of the Folk Theorem, see section 4).

In the continuum state space version of Okuno-Fujiwara and Postalewaite(1995) model, the games $\Gamma^{\infty}(\delta)$ consists of stage games which are defined as follows.

(1)*The set of players* The set of players is a finite set $I = \{1,2,\dots,n\}$.

(2)*The set of strategies available to player* i The set of strategies available to player i is a compact interval $A_i = [\underline{a}_i, \overline{a}_i]$ whose generic element is denoted by a_i.

(3)*The set of statuses of player* i The set of statuses of player i is a compact interval $X_i = [\underline{x}_i, \overline{x}_i]$ whose generic element is denoted by x_i. Suppose that this statuses of agents are determined by some rule according to outcome of play in the precedent period. This rule is described by a transition mapping $\tau_i : A \times X \to X_i$.

(4) *The payoff to player* i The payoff π_i to player i is represented by a real-valued function defined over the product space $A \times X_i$

In this setting, the status level x_i is attached to every player according to his strategy a_i in the previous period. If player i 's status level is x_i and its transition mapping τ_i is as in the definition and the distribution of other players status level is changed according to a transition function $P_{-i}(x_{-i}^{'}; a, x_{-i})$ over time, then a transition function of status levels in the whole society is given by

$$P(x'; a, x) = \begin{cases} P_{-i}(x_{-i}^{'}; a, x_{-i}) & if \quad \tau_i(a, x) = x_i' \\ 0 & if \quad \tau_i(a, x) \neq x_i' \end{cases}.$$

Given this, the agents' status levels in each period follow the following Markov process.

$$q_{t+1} = \int_X P(x'; a, x) q_t(dx')$$

Since per-period payoff π_i to player i is a function of strategies a taken by players and his status level x_i, his value function is of the following form.

$$v^{\infty}(x, a; \beta, p) = (1-\delta)\pi_i(x_i, a_i, a_{-i}) + \delta \int_X v^{\infty}(x', a_i, a_{-i}) q(x'; a, x) dx'$$

A solution in this game characterized by a Markov strategy is calculated by means of the ordinary dynamic programming. It is defined as follows.

Definition 2.1 (Norm Equilibrium Okuno-Fujiwara and Postlewaite (1995))

A triplet $(\beta^*, p^*) = (\tau^*, \sigma^*, p^*)$ is called a norm equilibrium of if

(a) p^* is stationary given β^*,

(b) for all $i \in I$, for all $x_i \in X_i$ and for all $a_i \in A_i$,

$$v^{\infty}\left(x,\sigma_i^*;\beta^*,p^*\right) \geq v^{\infty}\left(x,a_i;\beta^*,p^*\right)$$

Condition (a) requires that the status distribution in the equilibrium state is stationary. And condition (b) require that strategy profile constituting a norm equilibrium must satisfy an ordinary equilibrium condition.

As discussed above, the most striking feature of this model is that this analyzes the norm-guided behavior in the framework of equilibrium analysis. And this characterizes norm-guided behavior taken by players who pursue their own self-interest as an equilibrium behavior. In that sense, this reconciles two approaches discussed above. This, however, does not define a social norm itself as an equilibrium behavior. Instead, this introduce the concept of norm equilibrium which is attained under the guidance of social norm.

3. A Social Convention ; Philosophycal Study and Equilibrium Analysis

3.1 A Social Convention

In this section, we discuss how a convention has been interpreted by social scientists and how it is formalized by the equilibrium concept. We address the first question in 3.1 and second question in 3.2. In the case of a social convention, unlike the case of a social norm, there is no conflict between the interpretation of equilibrium analysis and of philosophycal study. Roughly speaking, both of them define it as a pattern of behavior which is spontaneously chosen by agents on the basis of their belief about what the others' will do. According to Lewis (1969), a convention is a pattern of behavior, that is customary, and self-enforcing. Everyone conforms, everyone expects others to conform, and everyone wants to conform given that everyone else conforms. In this interpretation, a social convention is the outcome learning in a repeated strategic interaction is ultimately a function of preference and expectation of agents.

To define it exactly, we use the following five concepts, i.e., rationality, inductive standard, background information, common knowledge and expectation. As for rationality, economists assume that agent's preference can be represented by a complete and transitive preference relation over possible outcomes.[4] They also assume that agents' inductive standard can be described by an optimization calculation and Baynesian updating. And they assume that agents have complete information or receive some public signal.

As shown in the above Lewis' definition convention is supported by high-order expectations, i.e., everyone conforms to the convention because he expects everyone else to conform to it because every one of them expects everyone else to conform to it, and so on. For these high-order expectation to be consistent with each other, rationality, inductive standard, background information of each agent should be common knowledge in the sense of Aumann (1976). That is, everyone knows that everyone else know about his rationality, inductive standard and background information and everyone else know that he knows that everyone else know that about his rationality, inductive standard and background information, and so on. And for them to be common knowledge, there must be some base for it. As simple examples of it, we can think of agreement, salience and precedent.

Here, let us introduce Lewis (1969)'s definition of social convention.[5]

Definition (Social Convention) A regularity in the behavior of members of a population when they are agents in a recurrent situation is a convention if and only if it is common knowledge in the population that, in almost all instance of the situation among members of the population,

(1) almost everyone conforms to it;

(2) almost everyone expects almost everyone else to conforms to it;

(3) almost everyone has the almost same preferences regarding all possible combinations of actions;

(4) almost everyone prefers that any one more conform to it , on condition that almost everyone conform to it;

(5) almost everyone prefers that any one more conform to some possible

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[4] Agent's preference relation is complete if any x, y $x \geq y$ or $x \leq y$. And it is transitive if $x \geq y$ and $y \geq z$ implies $x \geq z$.

[5] To be precise, Lewis (1969) provides the four definitions of social convention. And this is his final definition, see pp.78.

regularity in the behavior of the population in a situation, on condition that almost everyone conform to it. This regularity is such that almost no one in almost any instance of a situation among members of the population could conform both to a social convention and to it.

Third requirement is met in most cases. Agents' expectations about other agents' actions in second, forth and fifth requirements can be formed appropriately under the assumption of common knowledge. This implies that there must exist a convention supported as an equilibrium outcome based on a reasonable belief on other agents' choices in games. Note, however that convention should be strictly distinguished from norm because there is no sanction against not confirming to it. But convention is more formal than habit because convention is adopted by members of a society, not private one.

3.2 A Social Convention as an Equilibrium of Games

Young (1993) has proposed a game theoretical account of conventions according to which a convention is broadly defined as a Nash equilibrium. To analyze reasoning process of players, he uses adaptive process as follows. In the process, players are boundedly rational in that they observe limited size of sample of plays from recent time periods and they have only finite memory. And he defines a convention as an absorbing steady state supported as a strict Nash equilibrium in pure strategy in the game. Incompleteness of sample creates enough stochastic variability to prevent the process from becoming stuck in suboptimal cycles. Finite memory allows past miscoordinations to be forgotten forever.

In his analysis, he restricts attention to the long-run equilibrium selected in the presence of random perturbation. To this end, mutations who do not know nothing about past plays of the game are introduced every period. He shows that continual mutations which occur along the dynamic path at the individual level can select a risk dominant equilibrium as a long run equilibrium. This result is shown also by Kandori, Mailath and Rob (1993) and Ellison (1993) independently.

4. Folk Theorem and Repeated Games

In the following three sections, we discuss useful concepts for our analysis by which norms and conventions are characterized. By doing so, we present theoretical implications of norms and conventions as equilibria. First, the present section discusses the theory of repeated games and the most striking result in that field, i.e., the well-known "Folk theorem". Second, the next section discusses the theory of evolutionary games and the concept of "bounded rationality" in that context. Lastly, we discuss the concept of complementarity. The basic idea of repeated game theory is that people may behave quite differently toward those with whom they expect to have a long-term relationship from toward those with whom they expect no future interaction. And the Folk Theorem provides the rigorous foundation to this idea. The Folk Theorem states that if the single-period game is repeated infinitely, efficiency will be attained with supergame equilibria. Precisely, all feasible and individually rational payoffs can be attained as an equilibrium payoff. This result has been studied by many theorists under a variety of conditions, e.g., Friedman(1971), Benoit and Krishuna(1985), Fudenberg and Maskin(1986) and Abreu (1988). As usual, we begin our argument by presenting the framework of argument in this section.

Consider the repeated games $\Gamma^{\infty}(\delta)$ with stage games $\Gamma = \{I, A_i, \pi_i, \delta\}$ defined as follows.

(1) *The set of players* The set of players is a finite set $I = \{1,2,\ldots,n\}$

(2) *The set of strategies available to player i* The set of strategies available to player i is a finite set $A_i = \{a_i^1,\ldots a_i^m\}$ whose generic element is denoted by a_i.

(3) *The payoff to player i* The payoff π_i to player i is represented by a real-valued function defined over the product space $\prod_{i\in N} A_i$ whose generic element is denoted by $a = (a_1,\ldots a_n)$.

(4) *Common discount rate* $\delta \in (0,1)$

(5) *The set of feasible and individually rational payoffs* The set of feasible payoffs is defined by $F(A) \equiv \{\pi(a) | a \in a\}$ where the payoff vector is $\pi = (\pi_1,\ldots \pi_n)$. The minmax payoff of player i in Γ is

$v_i = \min_{a_{-i} \in A_{-i}} \max_{a_i \in A_i} \pi_i(a_i', a_{-i})$. The payoff vector is individually rational if for all i, $\pi_i \geq v_i$. Then the set of feasible and individually rational payoffs is defined by

$$D(A) \equiv \{\pi(a) \in F(A) | \pi_i \geq v_i \quad for \forall i \in N\}.$$

Note that player i can obtain at least v_i by switching to his best response and

$$\max_{a_i \in A_i} \pi_i(a_i, a_{-i}) \geq \min_{a_{-i} \in A_{-i}} \max_{a_i \in A_i} \pi_i(a_i, a_{-i}) = v_i$$

from which it follows that any equilibrium of Γ yields player i a payoff at least v_i.

Let s^k be the strategy profile at stage k with $s^k = (a_1^k, \ldots a_n^k)$ where superscript represents the stage and subscript represents players. The history available to player i is denoted by $h^k = (s_1, \ldots s_n)$ and the set of histories by H^k. And the strategy profile in supergame Γ^k is called behavior strategy and is a function from the set of histories to the set of pure strategies, i.e., $s_i^k : H^k \rightarrow A_i$. If games are with perfect recall, then we can restrict our attention to behavioral strategy by Khun's Theorem. And we define the solution concept in behavior strategy for supergame Γ^k.

Definition 4.1 (Subgame Perfect Equilibrium) The strategy profile s^k is a subgame perfect equilibrium of Γ^∞ if a^k is a Nash equilibrium of Γ^∞ for any $h^k \in H^\infty$.

Moreover, we define two sets of equilibrium payoffs in Γ^k corresponding to two equilibria, i.e. Nash equilibrium and subgame perfect equilibrium, as follows,

$$NE(\Gamma^k) \equiv \{\pi_i^* \in D(A) | \pi_i^*(s^*) = \max_{a_i \in A_i} \pi_i(s_i, s_{-i}^*) \quad for \forall i \in N \quad and \quad a_i \in A_i\}$$

and

$$SPE(\Gamma^k) \equiv \{\pi_i^* \in D(A) | \pi_i^{k*}(a^*) = \max_{a_i \in A_i} \pi_i^k(a_i, a_{-i}^*) \quad for \forall i \in N \quad and \quad h^k \in H^k\}.$$

In the Folk Theorem, a particular feasible and individually rational payoff can be supported as an equilibrium payoff by using a punishment on a deviator from the strategy inducing it as its payoff. For the purpose, we need to impose one of the following assumptions on games.

Assumption 4.1 $D(A)$ has a nonempty interior.

This assumption requires that for all d in $D(A)$ and all ε, there exists an element d' in $D(A)$ such that $\|d-d'\|<\varepsilon$, and d' is in the interior of $D(A)$. This is used to prove that a particular payoff can be supported as a perfect equilibrium payoff. In this case, the punishment which is used to deter a deviation must be a equilibrium strategy. Then if it is costly for the punisher to punish the deviator, he must get a reward. But the reward the punisher gets must not be a reward for the deviator. For the reason, the dimension of $D(A)$ must equal to the number of players n. This condition is called the full-dimensiomality condition.

Assumption 4.2 For all $i \in N$ there exists $e(i)$ in $E(\Gamma^1)$ such that $e(i) > v_i$.

This is used to prove that a particular payoff can be asymptotically supported as a Nash equilibrium payoff of finitely repeated games. To do so, profitable deviations at the end of the games must be excluded. There must be an equilibrium strategy which yields a higher payoff to players than their minimax level.

Assumption 4.3 For all $i \in N$ there exists $e(i)$ and $f(i)$ in $E(\Gamma^1)$ such that $e(i)^i > f(i)^i$, and $D(A)$ has an nonempty interior.

This is used to prove that a particular payoff can be asymptotically supported as a perfect equilibrium payoff of finitely repeated games. To this end both of the above ideas (1) playing an equilibrium strategy at the end of the game and (2) rewarding the punishers are employed. Then, this is the most restrictive assumption of all.

With the above preparation, we can present the ordinary Folk Theorem.

Definition 4.2 (the Folk Theorem 1.) For any normal form game, the set of subgame perfect outcomes (and, hence, the set of Nash outcomes) of the supergame Γ^{∞} with discounting converges the set of feasible and individually rational outcomes of the one-shot game as the discount tends to one, i.e.,

$$\lim_{\delta\to 1} SPE\left(\Gamma^{\infty}(\delta)\right) = D(A).$$

Definition 4.3 (the Folk Theorem 2.) For any normal form game, the set of subgame perfect of outcomes (and, hence, the set of Nash outcomes) of the k-fold repetition of Γ converges to the set of feasible and individually rational outcomes of the one-shot game as k tends to infinity, i.e.,

$$\lim_{k\to\infty} SPE\left(\Gamma^{k}(\delta)\right) = D(A).$$

Unfortunately, the well-known Folk theorems imply that, in general, the set of perfect equilibria is vast, all feasible and individually rational payoffs can be attained as an equilibrium payoff.

Friedman(1971) shows the Folk Theorem 1 by using the Cournot-Nash punishment after a single deviation. Benoit and Krishuna(1985) show that under the assumption 3 the Folk Theorem 2 holds for finite repetitions provided that the underlying game has at least 2 Nash equilibrium payoffs for each player. Fudenberg and Maskin(1986) show under the assumption 1 and the full-dimensionality condition that the Folk Theorem 1 holds. Abreu (1988) presents the simple strategy combination which supports any subgame perfect equilibrium path and yields any subgame perfect equilibrium payoff, called simple penal codes.

Moreover, this result holds under a variety of situations. Fudenberg, Kreps and Levine (1990) show that the Folk Theorem holds in the case where the games are played by long-run and short-run players. In this case every short-run player plays one-shot best response to the strategies of the long-run players. Then we can be reduced to the case with only long run players. Kandori (1992) shows that the Folk Theorem holds in the situation where a game is played by overlapping generations of players. Even if all players are finitely-lived and are replaced by successors, if there are sufficient interactions between one generations of players and the next and if the overlapping periods are long enough, every mutually beneficial outcome can be supported as a perfect equilibrium. As discussed above, Kandori (1992) and Okuno-Fujiwara and Postlewaite (1995) show that the Folk Theorem holds in the situation where players are paired with partners randomly drawn from a population to play the game and change their partners over time.

5. Evolutionary Games and Bounded Rationality

5.1 the Basic Framework and Static Justification of ESS

In this section, we introduce the solution concepts within the framework of evolutionary game and discuss the relationship between the ordinary solution concepts in non-cooperative game theory and them. Specifically, we first discuss the relationship between the concept of the evolutionary stable state (ESS) proposed by Maynard Smith(1982) and that of traditional non-cooperative equilibrium concepts, Nash equilibrium by (Nash(1950,1951)), perfect equilibrium by (Selten (1975)), and proper equilibrium by (Myerson (1978)). And then we present the equivalence of the limiting point of evolutionary process to ESS in a special case. This topic is concerning with the role that evolutionary process plays in the study of equilibrium refinement. That is, whether it can rule out intuitively implausible outcomes. First of all, we begin our discussion by presenting the basic framework of game in a conventional way. Here, we suppose simple two-player symmetric game consisting of the following components.

(1)*The set of players* The set of players is a finite set $I = \{1,2\}$

(2)*The set of strategies available to player* i The set of strategies available to player i is a finite set $A_i = \{a_i^1,...a_i^m\}$ whose generic element is denoted by a_i.

(3)*The payoff to player* i The payoff π_i to player i is represented by a real-valued function defined over the product space $\prod_{i=1,2} A_i$ whose generic element is denoted by $a = (a_1, a_2)$.

(4) *The set of mixed strategies of player* i The set of mixed strategies Δ_i

is the set of probability distributions over the set of pure strategies A_i whose elements sum to 1.

$$\Delta_i \equiv \{z_i : A_i \to [0,1] | \sum_{i=1}^{n} z_i = 1\}$$

In this setting, the average payoff to player i from choosing pure strategy a_i is given by

$$E_i(a_i, z_j) = \sum_{j=1}^{m} z_j \pi_i(a_i, a_j).$$

Then, the average payoff to player i from choosing mixed strategy z_i is given by

$$E_i(z_i, z_j) = \sum_{i,j=1}^{m} z_i \pi_i(a_i, a_j) z_j.$$

With this preparation, the first solution concept is introduced as follows.

Definition 5.1 (Evolutionary Stable Strategy (ESS))

A mixed strategy z_i is an ESS of the symmetric game if it satisfies the equilibrium condition

(1) if $z_i' \in \Delta_i$, then $E(z_i, z_j) \geq E(z'_i, z_j)$

(2) if $z_i \neq z'_i$ and $E(z_i, z_j) = E(z'_i, z_j)$, then $E(z_i, z'_j) > E(z'_i, z'_j)$.

Condition (1) requires that if z_i is an ESS, then z is a symmetric Nash equilibrium. And condition (2) requires that ESS z_i is a stable strategy against mutations. Note that there exist the following six underlying assumptions on games for defining ESS.[6]

(1) there is a large, random-mixing population
(2) this population is monomorphic
(3) there is asexual reproduction
(4) mixed strategies can exist and do breed true
(5) the individuals involve in pairwise contests only
(6) contest is symmetric and static

Evolutionary stability is essentially a condition that there be no profitable entry opportunities given the actions of the incumbent population. When pairs of players from a single population are randomly, repeatedly and anonymously paired to play a particular two person symmetric game, evolutionary stability provides a justification for equilibrium entirely

[6] See van Damme (1987)pp.218.

different from those advanced in traditional game theory.

The results of characterization of ESS are stated hereafter. First result is a characterization of ESS. This states that the strategy profile which is the only best response against itself must be ESS. In this case, the best response against itself for each player is singleton.

Fact 5.1 *If* (z_1, z_2) *is strict equilibrium, then* z_i *is an ESS.*[7]

The next result by Haigh (1975) determines the number of ESS. This states that the number of ESS is at most finite. However, This does not say that the set of ESS For a particular game is nonempty. A simple case where the set of ESS is nonempty is 2×2 symmetric matrix games with at least one strict equilibrium. As typical examples of interest, we can think of games with a dominant strategy equilibrium, pure coordination games and games with a unique symmetric equilibrium which is in mixed strategies.

Fact 5.2 *The number of ESS is finite (but possibly zero).*

Fact 5.3 *If* $[b_{kl}]$ *is a* 2×2 *fitness matrix, i.e.,*

$$\pi_i(a_1^1, a_1^2) = b_{11},\ \pi_i(a_1^i, a_2^j) = b_{12},\ \pi_i(a_2^i, a_1^j) = b_{21} \text{ and } \pi_i(a_2^1, a_2^2) = b_{22}$$

with $b_{11} \neq b_{21}$ *and* $b_{12} \neq b_{22}$, *then* $[b_{kl}]$ *has an ESS.*

With notations in this fact, the above examples are characterized as follows. Games with a dominant strategy equilibrium is defined by the inequalities $(b_{11} - b_{21})(b_{22} - b_{12}) < 0$ and pure coordination games by $b_{11} > b_{21}$, $b_{22} > b_{12}$ and games with a unique symmetric equilibrium which is in mixed strategies by $b_{11} < b_{21}$, $b_{22} < b_{12}$.

Next, we present the relation between ESS and the solution concepts of conventional game theory. First, we provide the relation between ESS and the perfect equilibrium proposed by Selten (1975). This is one of the most widely accepted refinements of Nash equilibrium. The basic idea behind the perfectness is that each player with a small probability makes mistake. This implies that every pure strategy is chosen with a small probability. The next

[7] See van Damme (1987) pp219.

result states that every ESS must be a perfect equilibrium. This implies that ESS is a refinement of Nash eqiulibrium.

Definition 5.2 (Perfect Equilibrium (Selten (1975)) An equilibrium z of Γ is a perfect equilibrium of Γ if z is a limit point of a sequence $\{z(\eta)\}_{\eta \downarrow 0}$ of perturbed strategies with

$$E_i(z(\eta)) \geq E_i(z(\eta) / z_i'(\eta))$$

for all η.

Fact 5.4 *If z_i is an ESS, then (z_1, z_2) is a perfect equilibrium.*

Second, we provide the relation between ESS and the proper equilibrium proposed by Myerson (1978). This is also a refinement of Nash equilibrium concept. And the relationship between these facts is provided by fact 5.6.

Definition 5.3 (Proper Equilibrium (Myerson (1978)) An equilibrium $z(\varepsilon)$ of Γ is an ε-proper equilibrium of Γ if $z(\varepsilon)$ is completely mixed and satisfies

if $E_i(z(\varepsilon)/k) \geq E_i(z(\varepsilon)/l)$, then $z_i^k(\varepsilon) \leq \varepsilon z_i^l(\varepsilon)$ for all i, k, l.

z is a proper equilibrium if z is a limit point of a sequence $\{z(\varepsilon)\}_{\varepsilon \downarrow 0}$ if $z(\varepsilon)$ is an ε-proper equilibrium of Γ.

Fact 5.5 *If z_i is an ESS, then (z_1, z_2) is a proper equilibrium.*

Fact 5.6 (Myerson (1978) *For any finite game in normal form, the set of proper equilibria is a nonempty subset of the set of perfect equilibria.*

Note that ESS is a solution concept in static games. For our purpose of analyzing formation and sustenance of convention and norm through the repeated interaction over time, we must justify the concept of ESS in dynamical setting. This is argued in the following section.

5.2 Dynamic Justification of ESS ; the Selection Dynamics

In this part, we discuss the relationship between ESS and the limiting point of evolutionary dynamic process. The results in this subsection and those in the previous subsection together show evolutionary process based on the principle that the fittest survive can select reasonable outcome from the game theoretical point of view. This is important from not only the theoretical but also practical point of view because this is concerning with explanatory power of evolutionary theory. First one is the problem of equilibrium refinement. This is the problem whether evolutionary process can rule out intuitively implausible outcomes and predict only plausible outcomes. To do so, let us define the evolutionary process whose steady states have relation with ESS formally.

The dynamical system considerd here is called selection dynamics (see Nachbar (1990), Samuelson and Zhang (1992)).

Definition 5.4 (Selection Dynamics)

For $\phi_i : \Delta_i \to \Delta_i$, the systems given by $\dot{z}_{it} = \phi_i(z_{it})$ are continuous selection dynamics if they satisfy the following three conditions.

(1) Lipschitz Continuity For any $z_i, z'_i \in \Delta_i$, there exists a real number k such that

$$|\phi_i(z_i) - \phi(z'_i)| \le k|z_i - z'_i|.$$

(2) Forward invariance If z_{it} is the solution path from $z_{i0} \in \Delta_i$, then

$$\phi_i(z_{it}) \ge 0 \text{ if and only if } z_{it} \ge 0.$$

(3) Relative monotonicity For any positive $z_i, z'_i \in \Delta_i$ with $z_{it} \ge z'_{it}$,

$$\frac{\phi_i(z_{it})}{z_{it}} \ge \frac{\phi_i(z'_{it})}{z'_{it}}.$$

Definition 5.5 (Steady State)

z_i^* is a steady state of dynamics $\dot{z}_{it} = \phi(z_{it})$ if it is a fixed point, i.e., $\phi(z_i^*) = 0$. And it is asymptotically stable if it has some neighborhood $N_\varepsilon(z_i^*)$ such that $z_{it} \to z_i^*$ as $t \to \infty$ whenever $z_{it} \in N_\varepsilon(z_i^*)$.

The relationship between the limit point of evolutionary process and ordinary Nash equilibrium was established for many models. This result is called "convergence or stability implies Nash" theorem and interpreted by Samuelson (1994) as a version of Folk theorem of evolutionary game. Nachbar (1990) shows that if a dynamic process satisfies monotonicity condition, then a limiting outcome of a converging process must be a Nash equilibrium for symmetric games. Samuelson and Zhang (1992) proved the same result for asymmetric games. Friedman (1990) proved the same result under the weaker assumption called compatibility. Here, we restrict our attention the results obtained for a special case of selection dynamics called replicator dynamics (hereafter RD. See Hofbauer and Sigmund (1988)). If the average payoffs from choosing a particular pure strategy a_i and from choosing a mixed strategy are given as in the previous subsection, then the RD are given by

$$\dot{z}_i = \lambda_i z_i \left[E(a_i, z_j) - E(z_i, z_j)\right].$$

As for the relationship between the limit point of the RD and ESS, the following result is obtained. This result is obtained by Hofbauer, Schuster and Sigmund (1979). This states that ESS is sufficient condition for the strategy profile to be asymptotically stable under the RD.

Fact 5.7 *Let z be an ESS, then the state is an asymptotically stable equilibrium of the RD.*

The following two facts provide the relationship between steady state of the RD and Nash equilibrium. The first result states that Nash equilibrium is sufficient for the strategy profile to be a steady state. But it is not necessary. The second fact presents the additional condition for the steady state to be Nash equilibrium.

Fact 5.8 *Let z defines a symmetric Nash equilibrium, then it is a steady state of the RD. The converse need not be true.*

Fact 5.9 *Let z be an asymptotically stable equilibrium of the RD, then z is a Nash equilibrium.*

The facts presented above are useful for us to characterize a social convention obtained as an equilibrium within the framework of evolutionary games. The reminder of this subsection, we briefly discuss the problem of equilibrium selection studied by Kandori, Mailath and Rob (1993), Young (1993) and Ellison (1993) and so on. This is the problem whether evolutionary process select a unique outcome in consistent ways. The key concepts for equilibrium selection are proposed by Harsanuy and Selten(1988). It is defined for the two strict equilibrium points.

Definition (the Risk Dominance Equilibrium) *A strict equilibrium* z *of* Γ *is risk dominates the other strict equilibrium* z' *of* Γ *if* z *satisfies*

$$E(z) - E(z / z_i) > E(z') - E(z'/z_i').$$

Kandori, Mailath and Rob (1993) show that continual mutations which occur along the dynamic path at the individual level can select risk dominant equilibrium as a long run equilibrium. And Young (1993), Ellison (1993) Their results inspire the study of equilibrium selection in the context of stochastic evolutionary game in the past five years.

5.3 Evolutionary Interpretation of Bounded Rationarity

As early as 1950's Simon (1955,1957) proposed the concept of "bounded rationality" . He hypothesized that economic agents perform limited searches and accept the first satisfactory decision rather than they perform exhaustive searches over all possible decisions and pick the best. This hypothesis was called "satisfying hypothesis" and had a large impact on economics. It, however, was not analyzed in a rigorous way. Instead, in 1970's, game theorists pursued the rationality of agents and proposed a variety of solution concepts requiring strong rationality to prescribe a plausible strategy, e.g., perfect equilibrium, Baynesian equilibrium and so on. Recently, game theorists have cast doubt on the assumption that economic agents are completely rational so that they can exploit any information about the whole structure and future state of the economy . And they formalize the idea by introducing the concepts of evolution and learning into game theory.

The interpretations of bounded rationality in the context of evolution or learning in game theory are as follows. [8]

(1) Players ignore strategic considerations regarding the future or do not take into account the long run implications of their strategy choices. Or, they ignore the fact that other players are also engaged in a dynamic learning process as they do *(myopia)*.

(2) Players are too naïve to perform an optimization calculation over all possible alternatives no matter how high the resulting expected future payoffs are *(naïve)* . Instead, they accept some satisfactory decision they can easily make.

(3) Players gradually learn opponents' behavior through repeated interaction, e.g., player's observation is imperfect. Even though they are rational players with perfect foresight there is significant inertia *(inertia)*, e.g. , changing one's strategy is costly.

Based on these interpretations, many learning models are provided, e.g., dynamical process with Darwinian property (Kandori, Mailath and Rob (1993) and Ellison (1993)), adaptive play with mistakes (Young (1993)) and learning models by Cannning (1992) and so on.

6. Strategic and Institutional Complementarity

In this section, we discuss the concept of "complementarity" which is the key concept in the study of the coordination of choices of behavior among agents. When institutional arrangements are chosen as equilibrium strategies which are mutually complementary in the social game, element institutions also become mutually complementary. This is defined by Aoki(1995) as institutional complementarity. Because of this, the economic system consisting of mutually complementary institutional arrangements may be difficult to change. Similarly, under the assumption that social conventions or norms are selected as mutually complementary equilibrium strategies through repeated interaction among a large number of agents, this

[8] For further discussion, see Kalai and Lehrer (1993) and Kandori, Mailath and Rob (1993).

is also difficult to change. However, this does not necessarily imply that only one kind of behavior pattern can survive in the long run. In a class of games where strategies of players are complements in the sense that maginal profitability of a player increases with the strategies of his opponents, is called a supermodular game, equilibrium is likely to be multiple. This explains the possibility of the diversity of behavior patterns across societies. Here, let us define the supermodular games in a formal way.

(1)*The set of players* The set of players is a finite set $I = \{1,2,\ldots,n\}$

(2)*The set of strategies available to player* i The set of strategies available to player i is a compact interval $A_i = [\underline{a}_i, \bar{a}_i]$ whose generic element is denoted by a_i. Suppose that A_i is complete lattice, i.e., it is a partially ordered set with a maximal element and a minimal element and for all nonempty subset $A'_i \subset A_i$, $\inf(A'_i) \in A_i$ and $\sup(A'_i) \in A_i$.

(3)*The payoff to player* i Suppose that payoff π_i to player i is supermodular , i.e.,

for all $a, a' \in A$, define $a \wedge a' = \inf\{a,a'\}$ and $a \vee a' = \sup\{a,a'\}$,

$$\pi_i(a) + \pi_i(a') \le \pi_i(a \wedge a') + \pi_i(a \vee a').$$

The following Characterization Theorem presents necessary and sufficient condition for a twice continuously differentiable function to be supermodular.

Fact 6.1 (the Topkis's Characterization Theorem) *Let* $I = [\underline{x}, \bar{x}]$ *be an interval in* R^n. *Suppose that* $f: R^n \to R$ *is twice continuously differentiable on some open set containing* I. *Then* f *is supermodular on* I *if and only if for all* $x \in I$ *and all* $I \ne j$, $\partial^2 f / \partial x_i \partial x_j \ge 0$.

The following Monotonicity Theorem states that the maximizer of supermodular function is monotone nondecreasing in a parameter.

Fact 6.2 (the Topkis's Monotonicity Theorem) *Let* S_1 *be a lattice and* S_2 *a partially ordered set. Suppose* $f(x,y): S_1 \times S_2 \to R$ *is supermodular in* x *for given* y *and has increasing differences in* x *and* y. *Suppose that* $y \ge y'$ *and that* $x \in M \equiv \arg\max f(x,y)$ *and* $x' \in M' \equiv \arg\max f(x,y')$. *Then* $\inf\{x,x'\} \in M'$ *and* $\sup\{x,x'\} \in M$. *In particular when* $y = y'$, *the set of maximizers of* f *is* *a sublattice.*

The following Fixed Point Theorem states that a monotone nondecrease function from a complete lattice to itself has at least one fixed point.

Fact 6.3 (the Tarski's Fixed Point Theorem) *If T is a complete lattice and $f: T \to T$ is a nondecreasing function, then f has a fixed point. Moreover, the set of fixed points of f has $\sup\{x \in T | f(x) \geq x\}$ as its largest and $\inf\{x \in T | f(x) \leq x\}$ as its smallest element.*

Second and third facts together ensure the existence of equilibrium in pure strategy in supermodular games. It, however, is not necessarily implies that equilibrium is unique. The following fact provides the upper and lower bounds of the set of equilibria.

Fact 6.4 *Let Γ be a supermodular game. Then there exists a pure Nash equilibrium. Moreover, there exist largest and smallest pure Nash equilibriua in the given order.*

The following fact is useful to evaluate an equilibrium point from the welfare point of view.

Fact 6.5 (the Welfare Theorem) *Let $\underline{a}_i$ and $\bar{a}_i$ be the smallest and largest elements of A_i and suppose a^* and $a^{*\prime}$ are two equilibria with $a^* \geq a^{*\prime}$.*
(1) If $\pi_i(\underline{a}_i, a_{-i})$ is increasing in a_{-i}, then $\pi_i(a^*) \geq \pi_i(a^{*\prime})$.
(2) If $\pi_i(\bar{a}_i, a_{-i})$ is decreasing in a_{-i}, then $\pi_i(a^*) \leq \pi_i(a^{*\prime})$.
If the condition in (1) holds for some subset of players N_1 and the condition in (2) holds for the reminder N_1 / N, then the largest equilibrium is the most preferred equilibrium for player in N_1 and the least preferred for the remaining players, while smallest equilibrium is least preferred by the players in N_1 and most preferred by the remaining players.

The following fact is a corollary of the Welfare Theorem and states that if the game has a unique equilibrium in pure strategy in pure strategy then the equilibrium is the Pareto-best. Although this is a desirable property of supermoduler game, existence of unique equilibrium in general requires strong assumption.

Fact 6.6 *Let Γ be a supermodular game. If the game Γ has a unique pure Nash equilibrium, then Γ is dominance solvable, i.e., all players are indifferent between all outcomes that survive the iterative procedure in which all weakly dominated strategies of each player are eliminated at each stage.*

Note that as pointed by Milgrom and Roberts (1990), if the game is a smooth supermodular game with continuously differential payoff function and an equilibrium point is interior of the set of strategies for all agents, even Pareto-best equilibrium is not Pareto optimum. As a simple example of such game, we can think of 2-person Cournot game where a Nash equilibrium remains after the iterated elimination of dominated strategies but it is not Pareto optimum.

7. Conclusions

And the outline of each section is as follows. Section 2 and 3 are devoted to comparison of interpretations of social convention and norm by economists and game theorists with that by other social scientists. A social convention and norm have been studied not only by economists but also other social scientists and there has been much difference between their definitions. Specifically, economists regards economic phenomena observed in the real economy as consequences of agents' choice subject to physical and institutional constraints, e.g., endowment, technology, market power and so on. And they interpret social conventions and norms in the same way. On the other hand, other social scientists define it as orientations by which individuals are guided. That is, they interpret them to be constraints on agents' decision-making. We call the former "equilibrium analysis" and the latter "philosophycal study".

Section 4 and 5 are devoted to presentation of two analytical tools which play the key role in our analysis. First one is the Folk Theorem in the theory of repeated games. This states that efficient cooperation can be supported as an equilibrium outcome in non-cooperative games.

Second one is the concept of evolutionarily stability in the theory of evolutionary game. The fact that evolutionarily stable equilibrium is Nash equilibrium is called by Samuelson and Zhang (1992) and Samuelson (1994) "stability or convergence implies Nash theorem" and a version of the Folk Theorem of evolutionary games.

Section 6 is devoted to presentation of the concept of "complementarity". Complementarity between strategic choices of agents has been found in the Japanese management system, e.g., "lifetime employment" and "enterprise unionism" in the employer-employee relationship, "seniority pay" in wage structure and promotion system and "mutual stockholding" in corporate governance. Then, this is the key concept for our further study of Japanese economy.

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